



a fingerprint school



MAX. MARKS: 80

DATE: 09/03/2021

MODEL EXAMINATION-3

GRADE: 12

MATHEMATICS (041)

TIME: 3 HOURS

General Instructions:

- 1. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks .
- 2. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions .
- 3. Both Part A and Part B have choices.

Part-A

- 1. It consists of two sections- I and II.
- 2. Section I comprises of 16 very short answer type questions.
- 3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part-B

- 1. It consists of three sections- III, IV and V.
- 2. Section III comprises of 10 questions of 2 marks each.
- 3. Section IV comprises of 7 questions of 3 marks each.
- 4. Section V comprises of 3 questions of 5 marks each.
- 5. Internal choice is provided in 3 questions of Section –III, 2 questions of Section IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART-A SECTION-I

All questions are compulsory. In case of internal choices attempt any one.

1. Find the direction cosines of the normal to YZ-plane.

OR

If *A* is a square matrix of order 3 and |A| = 5, then find the value of |2A'|.

2. Write the vector equation of the line passing through the point (-1, 5, 4) and perpendicular to the plane z = 0.

3. The number of arbitrary constants in the particular solution of a differential equation of second order is _____

4.
$$\int_{\frac{-3\pi}{5}}^{\frac{3\pi}{5}} \cos ec^2 x \, dx =$$
 OR Evaluate: $\int_{0}^{1} y(1-y)^5 dy$.

5. The length of the perpendicular drawn from the point (4, -7, 3) on y axis is

6. Find
$$\int e^{x} (1 + \tan x + \sec^2 x) dx$$
. OR

Find the coordinates of the point where the line $\frac{x+3}{3} = \frac{y+1}{1} = \frac{z-5}{-5}$ cuts XY-plane.

- 7. Determine the order and degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + y = \frac{dx}{dy}$
- 8. For what value of *n* is the following a homogeneous differential equation $\frac{dy}{dx} = \frac{x^3 + y^n}{x^2 y + xy^2}.$
- 9. Find the length of the intercept, cut off by the plane 2x + y z = 5 on the x -axis. OR

Find the direction cosines of the normal to XY-plane.

10. If A and B are independent events such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and P(B) = p, then find the value of p. OR Let $A = [a_{ij}]$ be a square matrix of order 3 x 3 and |A| = -5. Find the value of

 $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$, where A_{ij} is the cofactor of element a_{ij}

11. Find the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$. 12. Find the area of the region bounded by the curve $y^2 = x$ and the lines x = 1 and x = 4 and the x axis in the first quadrant.

13. If
$$A = \begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}$$
 and $B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, find $2A + B$

14. A random variable has the following distribution :

X : -1 0 1 2

P(X): 1/3 1/6 1/6 1/3 Does it represent a probability function?

- 15. Three coins are tossed. Find the probability of getting at least two tails.
- 16. Solve: $\frac{dy}{dx} = \frac{y}{x}$

Section II

Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question .Each question carries 1 mark

17. The tunnel is in the shape of a parabola with a span 100 feet and highest point on the parabola is 10 feet above the road.



(I). The equation of the parabolic tunnel is

(a)
$$x^2 = 250y$$
 (b) $x^2 = -250y$ (c) $y^2 = 250x$ (d)
 $x^2 = 250$

(II). The value of the integral $\int_{-50}^{50} \frac{x^2}{250} dx$ is

(a) 1000/3 (b) 2500/3 (c) 250/3 (d) 0

(III). The integrand of the integral $\int_{-50}^{50} \frac{x^2}{250} dx$ is ______ function

(a) even (b) odd (d) neither even nor odd (d) can't be determined (IV). The area formed by the curve $x^2 = 250y$, x-axis, y=0 and y=10 is

(a)
$$\frac{1000\sqrt{2}}{3}$$
 (b) 4/3 (c) 1000/3 (d) 0

(V). The latus rectum of the parabola $x^2 = 250y$ is

(a) x = 250 (b) 2x = 125 (c) x = 125/4 (d) y = 125/2

18. In a bank, principal increases continuously at the rate of 5% per year. Initially Rs.1000 is deposited in a bank.

(I). If P is the principal at any time t. The corresponding differential equation is

(a)
$$\frac{dP}{dt} = 5P$$
 (b) $\frac{dP}{dt} = \frac{P}{20}$ (c) $\frac{dP}{dt} = P$ (d) $\frac{dP}{dt} = \frac{20}{P}$

(II). The solution of the differential equation $\frac{dP}{dt} = \frac{P}{20}$ is

(a)
$$P = k e^{\frac{t}{20}}$$
 (b) $P = k e^{\frac{t}{5}}$ (c) $\log P = k e^{\frac{t}{20}}$ (d) none

(III) The value of P when t = 0, is

(a) P = 100 (b) P = 1000 (c) 5000 (d) 2500

(IV) In how many years Rs.1000 double itself?

(a) $20\log_5$ (b) $20\log_e 2$ (c) $2\log 2$ (d) none

PART-B

SECTION-III

19. If $A = \begin{pmatrix} 2 & -1 \\ 4 & 2 \end{pmatrix}$, then find A^{-1} . OR

Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find the values of a and b

20. Evaluate:
$$\int \frac{1}{\sin^2 x \cos^2 x} dx$$

21. Evaluate:
$$\int_{0}^{\pi/2} \frac{dx}{1+\sin x}$$
 OR $\int_{-5}^{5} |x-2| dx$

22. Find the value of the integral
$$\int_{0}^{2\pi} \cos^5 x \, dx$$

23. Find the area of the region bounded by the parabola $y^2 = 16x$ and its latus rectum.

24. Solve:
$$\log\left(\frac{dy}{dx}\right) = ax + by$$
 OR

Find the integrating factor of the differential equation

$$(x^{2}+1)\frac{dy}{dx} + 2xy = \frac{1}{(x^{2}+1)}$$

- 25. Show that the points with position vectors $\vec{a} 2\vec{b} + 3\vec{c}, -2\vec{a} + 3\vec{b} \vec{c}$ and $4\vec{a} 7\vec{b} + 7\vec{c}$ are collinear.
- 26. Find the value of *p* for which the vectors $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are (i) perpendicular (ii) parallel
- 27. Find the Cartesian equation of a line through A(3,4,-7) and B(1,-1,6).
- 28. Find the distance of the point $2\hat{i}+\hat{j}-\hat{k}$ from the plane $\vec{r} \cdot (\hat{i}-2\hat{j}+4\hat{k})=9$

SECTION-IV

All questions are compulsory. In case of internal choices attempt any one.

29. Evaluate:
$$\int_{0}^{\pi/2} \log \cos x \, dx \quad \mathbf{OR} \quad \text{Evaluate:} \quad \int_{0}^{3} \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} \, dx$$

30. Solve: $(x^2 - y^2)dx + 2xy dy = 0$; given that y = 1 when x = 1. OR Solve: $(\tan^{-1} x - y)dx = (1 + x^2)dy$

31. Find the equation of the plane passing through the point $\hat{i} + \hat{j} + \hat{k}$ and parallel to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 5$ OR

Find the vector equation of the plane at a distance of 8 units from the origin and which is normal to the vector $2\hat{i} + \hat{j} - \hat{k}$

32. Solve the following graphically:

Maximize: Z = 5x + 3y

Subject to

$$3x + 5y \le 15$$

 $5x + 2y \le 10$

 $x \ge 0, y \ge 0$

33. A die is thrown three times. Events A and B are defined as below:

A: 4 on the third throw

B: 6 on the first and 5 on the second throw

Find the probability of A given that B has already occurred.

34. Three coins are tossed simultaneously. Consider the event E ' three heads or three tails', F ' at least two heads' and G ' at most two heads'. Of the pairs (E, F), (E, G) and (F, G), which are independent? which are dependent?

35. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bys and scooter respectively, but if he comes by other means of transport, then he will not be later. When he arrives, he is late. What is the probability that he comes by train?

SECTION-V

All questions are compulsory. In case of internal choices attempt any one.

36. If $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$, find A^{-1} . Use it to solve the equations

2x-3y+5z = 11, 3x+2y-4z = -5, x + y - 2z = -3.

- 37. Find the area of the region bounded by the curve $x^2 = 4y$ and the straight line x = 4y 2.
 - OR

Find the area of the region bounded by the curve $y = \sin x$ between x = 0and $x = 2\pi$

38. Find the equation of the plane passing through the points (2,1,-1) and A(-1,3,4) and perpendicular to the plane x-2y+4z=10. OR Find the equation of the plane through the line of intersection of $\vec{r}.(2\hat{i}-3\hat{j}+4\hat{k})=1$ and $\vec{r}.(\hat{i}-\hat{j})+4=0$ and perpendicular to $\vec{r} \cdot (2\hat{i}-\hat{j}+\hat{k})+8=0$.